ACALANES UNION HIGH SCHOOL DISTRICT
COURSE OF STUDY: CURRICULAR AREA – MATH

COURSE TITLE:	PRE CALCULUS HONORS
GRADE LEVEL:	11-12
COURSE LENGTH:	One Year
PREFERRED PREVIOUS COURSE OF STUDY:	Algebra 2 Advanced
CREDIT:	10 Credits
UC/CSU CREDIT:	Meets UC/CSU credit for mathematics requirement; subject area (“c”) with Honors designation
GRADUATION REQUIREMENT:	Fulfills 10 credits of mathematics credit (2 semesters beyond Algebra 1) required for graduation
STANDARDS AND BENCHMARKS:	California State Standards
ADOPTED:	May 2, 2018
INSTRUCTIONAL MATERIALS:	Pre-Calculus with Trigonometry: Concepts and Applications; Foerester; Kendull Hunt, 2012 Pre-Calculus Mathematics for Calculus; Stewart, Redlin, Watson; Cengage Learning, 2016

COURSE DESCRIPTION: Pre-Calculus combines many of the trigonometric, geometric and algebraic techniques needed to prepare students for the study of calculus and strengthens their conceptual understanding of problems and mathematical reasoning in solving problems. This course takes a functional point of view toward these topics. Students will be introduced to matrices, vectors, polar graphing, parametric equations, conic sections, and introductory calculus topics.

COURSE OBJECTIVES: Content Objectives

During the course the students will:

1. Extend knowledge of solving and graphing algebraic, exponential, and logarithmic functions.
2. Explore and apply properties of trigonometric and periodic functions to graph and solve real world applications.
3. Investigate analytic geometry topics such as non-Cartesian graphing (eg. conics, polar, vectors, and complex) and parametric equations.
4. Apply the concepts of matrix manipulation to analyze and solve linear algebra problems.
5. Analyze discrete and continuous concepts of mathematics.
6. Develop knowledge of introductory calculus topics.

During the course students will work toward:

1. Active learning through investigation and conjecture
2. Attending to precision and persevering in problem solving
3. Consistent study habits, organization, and personal responsibility for learning
4. Proficiency in writing about mathematics and fluency in using mathematical vocabulary
5. Strategic implementation of instructional technologies
6. Effective communication and collaboration

ASSESSMENT: Assessments are designed to promote and evaluate mathematical thinking. Teachers use engaging activities that involve students in investigating, conjecturing, verifying, applying, evaluating, and communicating in various assessment modalities.

Formal and informal assessments can be made on the basis of both individual and group work. Assessments should be from a variety of means and could include performance tasks, quizzes, tests, projects, investigations, and daily assignments.

Assessments should be measuring the following claims:

Claim #1 – Concepts & Procedures

Students can explain and apply mathematical concepts, as well as interpret and carry out mathematical procedures with precision and fluency.

Claim #2 – Problem Solving

Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.

Claim #3 – Communicating Reasoning

Students can clearly and precisely construct viable arguments to support their own reasoning and critique the reasoning of others.

Claim #4 – Modeling and Data Analysis

Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

Performance Tasks

Performance tasks are used to better measure capacities such as depth of understanding, research skills, and complex analysis, which cannot be adequately assessed with selected- or constructed-response items. These tasks may require students to evaluate, optimize, design, plan, model, transform, generalize, justify, interpret, represent, estimate, and calculate solutions. Performance tasks can be used for a variety of purposes such as topic engagement, formative assessment and evaluative assessment. They may be performed individually or in small groups, depending on the purpose of the assessment.

Performance tasks should:

- Integrate knowledge and skills across multiple claims
- Require student-initiated planning and management of information and ideas
- Reflect a real-world task and/or scenario-based problem
- Allow for multiple approaches
- Represent content that is relevant and meaningful to students
- Be assessed using an understandable rubric that provides meaningful feedback for students and the teacher

GRADING GUIDELINES: See AUHSD Grading Guidelines: Final Mark Rubric and Final Course Mark Determination Components

COURSE CONTENT:

Mathematical Practices

The Standards for Mathematical Practice are "habits of the mind of mathematically proficient students". They describe the attributes that mathematics educators at all levels are striving to develop in their students, as these practices are based on key mathematical processes and proficiencies. The goal of implementing these practices is to develop students who can use their knowledge of mathematics in flexible, sophisticated, and relevant ways across multiple disciplines.

#1 Make sense of problems and persevere in solving them (Hypothesize & Strategize)

- Students are:
  - Making conjectures about what the problem is asking and how they can begin to solve it
  - Checking for the reasonableness of the strategy as the work progresses and making adjustments as needed
- Teachers develop this skill by having students:
  - Explain the meaning of the problem and/or restate the problem
  - Analyze the given information and develop entry points into the problem and develop strategies for solving the problem
  - Execute and evaluate multiple strategies

#2 Reason abstractly and quantitatively (De/Contextualize)

- Students are:
  - Determining what numbers and symbols represent through the use of diagrams, graphs or equations
- Teachers develop this skill by having students:
  - Move between multiple representations to determine the meaning behind quantities
  - Express purely mathematical expressions with real world context and taking quantities out of context and representing them as abstract mathematical ideas or expressions

#3 Construct viable arguments; critique others' reasoning

- Students are:
  - Justifying their thinking by providing evidence based on mathematical properties and using that evidence to evaluate the reasoning of others
- Teachers develop this skill by having students:
  - Make conjectures, compare and contrast methods, and identify flawed logic by providing counter-example

#4 Model with Mathematics

- Students are:
  - Interpreting and constructing graphs, tables, number lines, diagrams or equations to model real-world situational data
- Teachers develop this skill by having students:
  - Use models to make interpolative and extrapolative inferences
  - Examine the effectiveness and appropriateness of a model

#5 Use appropriate tools strategically

- Students are:
  - Selecting appropriate math tools and technology to help solve problems including manipulatives, graphing utilities, tables, [matrices], computer applications, compasses, etc.
- Teachers develop this skill by having students:
  - Identify the strengths and weaknesses of different tools in relation to solving a given problem and also use tools to explore, confirm or deepen understanding

#6 Attend to Precision

- Students are:
  - Calculating quantities accurately through proper rounding (based on context), labeling of units of measure, and checking their work
  - Selecting a problem solving method that allows for appropriate precision
- Teachers develop this skill by having students:
  - Formulate precise explanations of their work using vocabulary and justify their rounding process
  - Re-examine their work or thinking process, and then demonstrate the method by which they check their answers

#7 Look For and Make Use of Structure

- Students are:
  - Looking for patterns or relationships and using that structure to simplify complex ideas
- Teachers develop this skill by having students:
  - Extend prior knowledge of similar situations to novel ones or break down complex problems in smaller parts which resemble simpler, more familiar ideas

#8 Look for and express regularity in repeated reasoning (Generalize)

- Students are:
  - Developing general methods, rules, or short cuts and determining when they are appropriate
- Teachers develop this skill by:
  - Facilitating activities which allow for students' "aha!" moments and then helping them use it to develop "rules" based on repeated trials with a process

The following table outlines all content standards and learning objectives taught in the Precalculus Honors course.

Common Core State Standard (CCSS)	Learning Objective
The Complex Number System (N-CN)	
Perform arithmetic operations with complex numbers.	
3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.	● Use conjugates to rationalize complex numbers. ● Simplify complex numbers.
Represent complex numbers and their operations on the complex plane.	
4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.	● Graph complex numbers in both Cartesian and polar form. ● Understand why the rectangular and polar forms of a given complex number represent the same number.
5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $\left(-1+\sqrt{3}i\right)^3 = 8$because $\left(-1+\sqrt{3}i\right)$has modulus 2 and argument 120°.	● Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane.

6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	● Calculate the distance between numbers in the complex plane and the midpoint of the segment.
Vector and Matrix Quantities (N-VM)	
Represent and model with vector quantities.	
1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g. $\boldsymbol{v}$, $\lvert\boldsymbol{v}\rvert$, $\lVert\boldsymbol{v}\rVert$).	● Identify the magnitude and direction of a vector quantity.
2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.	● Subtract vectors in component form.
3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.	● Solve real life problems involving vectors.
Perform Operations on Vectors	
(+) Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.	● Add and subtract vectors using different methods and understand the result. ● Find the resultant vector given magnitude and direction. ● Represent v as the additive inverse of vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction $v - w$as $v + (-w)$, where $-w$is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.	
(+) Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g. as $c\ (v_x, v_y) = (cv_x, cv_y)$. b. Compute the magnitude of a scalar multiple cvusing $\|\|\boldsymbol{cv}\|\| = \|\boldsymbol{c}\|\boldsymbol{v}$. Compute the direction of cv knowing that when$\|\boldsymbol{c}\|\boldsymbol{v} \neq \boldsymbol{0}$, the direction of cv is either along v(for $c > 0$) or against v(for $c < 0$).	• Represent scalar multiplication graphically by dilating vectors and reversing their direction. • Perform scalar multiplication component-wise. • Determine the magnitude and direction of a scalar multiple. • Interpret the graphical meaning of positive and negative scalars.
Perform operations on matrices and use matrices in applications.	
6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.	• Use matrices to solve real-world applications.
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.	• Multiply matrices by scalars.

8. (+) Add, subtract, and multiply matrices of appropriate dimensions.	● Determine when it is possible to add, subtract and multiply matrices. ● Add, subtract and multiply matrices.
9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.	● Correctly apply the commutative, associative and distributive property for matrices.
10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is non-zero if and only if the matrix has a multiplicative inverse.	● Understand the concepts of matrix multiplicative and additive identities and inverses.
11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.	● Understanding how to multiply vectors by using matrix multiplication. ● Work with matrices as transformations of vectors.
12. (+) Work with 2x2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.	● Find the area of a figure on a coordinate system using the determinant.

Seeing Structure in Expressions (A-SSE)	
Interpret the structure of expressions.	
1. Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of *P* and a factor not depending on *P*.	● Interpret various parts of equations or expressions and understand their meaning. ● Identify the graphical changes that occur when certain quantities in algebraic equations change.
2. Use the structure of an expression to identify ways to rewrite it. For example, see x^4-y^4 as $(x^2)^2-(y^2)^2$ thus recognizing it as a difference of squares that can be factored as $(x^2-y^2)(x^2+y^2)$.	● Use algebraic techniques to factor expressions. ● Recognize when an expression or equation can be re-written in a form that is more convenient to use.
Algebra with Polynomial and Rational Expressions (A-APR)	
Rewrite rational expressions.	
6. Rewrite simple rational expressions in different forms; write *a(x)/b(x)* in the form *q(x)+r(x)/b(x)*, where *a(x), b(x), q(x)*, and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x)*, using inspection, long division, or, for the more complicated examples, a computer algebra system.	● Divide polynomials using long and synthetic division. ● Simplify rational expressions. ● Express polynomials as the sum of a remainder and a quotient. ● Find excluded values of rational expressions, and state domain and range values.

7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a non-zero rational expression; add, subtract, multiply, and divide rational expressions.	● Simplify rational functions before performing addition and subtraction. ● Perform operations on rational expressions. ● Use factoring to simplify rational functions and to identify resulting domain restrictions.
Creating Equations (A-CED)	
Create equations that describe numbers or relationships.	
1. Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	● Solve problems involving quadratic, linear, rational, and absolute value equations. ● Create equations that represent one real world quantity and use algebraic techniques to simplify and solve.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	● Create and graph equations that represent two different real world quantities. ● Understand the meaning of slope, concavity, maximum and minimum values, and asymptotes of a graph.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.	● Interpret the end behavior of functions. ● Create reasonable domain restrictions to model specific situations.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.	● Determine which variable to isolate when given specific inputs and outputs. ● Use algebraic operations to factor and isolate variables.

Reasoning with Equations and Inequalities (A-REI)	
Solve systems of equations.	
8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.	● Articulate the efficacy of representing systems of linear equations in terms of coefficients in an array. ● Represent a linear system as a matrix equation. ● Connect operations previously made with systems of two equations to matrix operations with three or more variables, including addition and scalar multiplication. ● Apply matrix operations to solve systems of linear equations. ● Understand dimensional restrictions as they apply to matrix operations.
9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimensions of 3x3 or greater)	● Create inverse matrices for up to 2 x 2 matrices using determinants, and learn how to use technology for finding inverses for larger matrices. ● Use inverse matrices to solve linear systems.
Interpreting Functions (F-IF)	
Interpret functions that arise in applications in terms of the context.	
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.	● Given the graph of a function, identify the function type and find the particular equation algebraically. ● Identify intervals on which functions are increasing, decreasing, or constant. ● Determine the average rate of change of a function. ● Interpret intercepts within the context of an application problem. ● Use limits to describe the end behavior of functions. ● Find and interpret relative maximum and minimums in terms of the given quantities.

	● Given a verbal description of a periodic phenomenon write an equation using the sine or cosine function and use the equation as a mathematical model to make predictions and interpretations about the real world.
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h gives the number of person-hours it takes to assemble engines in a factory, then the positive integers would be an appropriate domain for the function.	● Define and interpret domain in a real world context.
Analyze functions using different representations.	
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	● Model real world data with polynomial functions. ● Graph a function and identify zeros, asymptotes, end behavior, and other key features. ● Use algebraic techniques to find the key features of a graph. ● Graph trigonometric functions using midline, amplitude, period, and phase shift.
10. (+) Demonstrate an understanding of functions and equations defined parametrically and graph them.	● Explore the functionality of parametric equations with regard to modeling where a parametric variable is useful. ● Eliminate the parameter to see the relationship to non-parametric functions and identify limitations on domain and range. ● Graph parametric equations by creating a table of values and by eliminating the parameter.

11. (+) Graph polar coordinates and curves. Convert between polar and rectangular coordinate systems.	• Identify the function equivalent in polar form, especially polar forms of conic sections. • Convert between Cartesian and polar equations. • Correlate domains and ranges between the two different systems of graphing.
Building Functions (F-BF)	
Build new functions from existing functions.	
3. Identify the effect on the graph of replacing by $f(x)$ by $f(x)+k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	• Interpret the graphical changes that occur when a function is transformed. • Use a graph to identify the transformation. • Graph a function using transformations of a parent function.
4. Find inverse functions. b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain.	• Verify by composition that one function is the inverse of another. • Find the values of an inverse function from a graph or table of the original function. • Restrict the domain of a non-invertible function to find the inverse.

Trigonometric Functions (F-TF)	
Extend the domain of trigonometric functions using the unit circle.	
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	● Understand how the unit circle translates to the graph of a trigonometric function.
Model periodic phenomena with trigonometric functions.	
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.	● Restrict the domain of a trigonometric function in order to find the inverse.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.	● Solve trigonometric equations for angles in problems modeling real world contexts.
Prove and apply trigonometric identities.	
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	● Derive the trigonometric formulas for angle addition and subtraction. ● Apply angle addition and subtraction formulas in solving problems. ● Use angle addition and subtraction formulas to verify trigonometric identities.
10. (+) Prove the half angle and double angle identities for sine and cosine and use them to solve problems.	● Derive the double and half angle trigonometric formulas. ● Apply double and half angle formulas in solving problems. ● Use double and half angle formulas to verify trigonometric identities.
Similarity, Right Triangles and Trigonometry (G-SRT)	

Apply trigonometry to general triangles.	
9. (+) Derive the formula $A = \frac{1}{2}ab\sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side	• Derive the formulas for the area of a non-right triangle.
10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.	• Derive the Law of Sines and Cosines.
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	• Apply the Law of Sines and Cosines to solve problems and real-world applications.
Expressing Geometric Properties with Equations (G-GPE)	
Translate between the geometric description and the equation for a conic section.	
3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.	• Define ellipses and hyperbolas using their constant sum and difference properties. • Use the characteristics of the equations to graph ellipses and hyperbolas.
3.1 Given a quadratic equation of the form $ax^2+by^2+cx+dy+e=0$, use the method for completing the square to put the equation into standard form; identify whether the graph of the equation is a circle, ellipse, parabola, or hyperbola and graph the equation.	• Transform between general form and standard form of conic section equations. • Identify the type of conic section from the equation and the graph.